Open Problem: AnsProlog encoding of win and lose (Chs 2,4)

The goal here is to develop an AnsProlog program that has the same characterization of winning and losing as the following logic program with respect to the well-founded semantics.

$win(X) \leftarrow move(X,Y), \textbf{not}\ win(Y).$
an arbitrary set of 'move' facts.

We will now give another characterization of this which does not appeal to the well-founded semantics.

1. We have a set of nodes.
2. We are given a set of facts about a binary predicate *move*. Intuitively, $move(a,b)$ means that there is an available move from node a to node b.
3. A strategy S is a function from nodes to nodes such that $S(X) = Y$ only if move(X,Y) is true.
4. Given two strategies S_1 and S_2, and a node a we define the trajectory followed by alternatively applying S_1 and S_2 as: $traj(a, S_1, S_2) = X_0^{a,S_1,S_2} X_1^{a,S_1,S_2} \ldots$, where

   $X_0^{a,S_1,S_2} = a$
   $X_{k+1}^{a,S_1,S_2} = S_1(X_k^{a,S_1,S_2})$ if k is even
   $= S_2(X_k^{a,S_1,S_2})$ if k is odd
5. The length of a trajectory $X_0 X_1 \ldots X_k$ is k.
6. A node a is said to be a winning node if there exists S_1 such that for all S_2 the sequence $traj(a, S_1, S_2)$ terminates and its length is odd.

7. A node a is said to be a losing node if for all S_1 there exists S_2 such that the sequence $traj(a, S_1, S_2)$ terminates and its length is even.

Question 1: Write an AnsProlog program Π whose answer set semantics corresponds to the *win* and *lose* above. (The solution to this is known.)

Question 2: Write an AnsProlog program Π which has a unique answer set corresponding to the *win* and *lose* above. (To the best of my knowledge, this is an open problem.)

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